Complete Knowledge: Preventing Encumbrance of Cryptographic Secrets

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***Abstract*—Most cryptographic protocols model a player's knowledge of secrets in a simple way. Informally, the player knows a secret in the sense that she can directly furnish it as a (private) input to a protocol, e.g., to digitally sign a message.**

The growing availability of Trusted Execution Environments (TEEs) and secure multiparty computation, however, undermines this model of knowledge. Such tools can *encumber* a secret sk and permit a chosen player to *access* sk *conditionally, without actually* knowing sk. By permitting selective access to sk by an adversary, encumbrance of secrets can enable vote-selling in cryptographic voting schemes, illegal sale of credentials for online services, and erosion of deniability in anonymous messaging systems.

Unfortunately, existing proof-of-knowledge protocols fail to demonstrate that a secret is unencumbered. We therefore introduce and formalize a new notion called *complete knowledge* (CK). A proof (or argument) of CK shows that a prover does not just know a secret, but also has fully unencumbered knowledge, i.e., unrestricted ability to use the secret.

We introduce two practical CK schemes that use special-purpose hardware, specifically TEEs and off-the-shelf mining ASICs. We prove the security of these schemes and explore their practical deployment with a complete, end-to-end prototype that supports both. We show how CK can address encumbrance attacks identified in previous work. Finally, we introduce two new applications enabled by CK that involve proving ownership of blockchain assets.

I. Introduction

Most cryptographic protocols are designed under a simple model of knowledge. If a player $\mathcal{P}$ knows a secret value sk, then she can explicitly furnish it as a (private) protocol input. In a digital signature scheme, for example, $\mathcal{P}$ inputs a private key sk to a locally executed algorithm to sign a message.

This basic, intuitive model of knowledge, however, can break down when sk is not controlled by a single player, but an *interactive functionality*. For example, sk might be stored exclusively in a trusted execution environment (TEE) such as Intel SGX [47], [48], AMD SEV [9], or AWS Nitro Enclaves [8]. The TEE could then *encumber* $\mathcal{P}$'s access to sk, by only allowing selective use. For instance, in the previous digital signatures example, a TEE could generate (and store) a private signing key sk for a user Alice, but only allow Alice to sign messages approved by an adversary.

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Such encumbrances can also be realized by multi-party computation (MPC) [36], [66] over sk among a committee that restricts its use. Encumbrance of secrets can undermine security in many cryptographic protocols, as we will show.

It may seem counterintuitive that a user / prover might *want* to encumber her own secret sk. Encumbrance of secrets, it turns out, can paradoxically *benefit* a user. For example, as highlighted in [30], [54], voters that choose to encumber secret keys used in a voting scheme can sell their votes to an adversary trying to subvert an election. Here, Alice might encumber her voting key sk so that she can only sign a ballot with candidate Bob, the choice of adversary Mallory. Alice can then sell Mallory an enforceable promise that if she votes, she will vote only for Bob. Remarkably, even techniques that specifically aim to prevent such vote-selling—e.g., so-called coercion-resistant voting schemes [27], [28], [41]—fail in the presence of such key encumbrance.

In this paper, we introduce and explore a new notion of knowledge called *complete knowledge* (CK). Complete knowledge embodies a strong notion of possession meant to rule out encumbrance of e.g., the secret key. Complete knowledge by a prover $\mathcal{P}$ of a secret sk means, informally, that it has *unencumbered* access to sk and thus *can use it for any desired purpose*, e.g., can sign any message of her choice.

CK can be leveraged in our voting example, by requiring Alice to prove that she has complete knowledge of her secret key sk before allowing her to vote. Here, CK would imply that Alice can always cast any desired vote and therefore, cannot sell Mallory an enforceable promise to vote only for Bob.

Our goals in this work are to formalize CK, implement it end-to-end, and shed light on its various applications.

The problem with proofs of knowledge. To understand CK, we build on the classical formalism of *proofs of knowledge* (PoKs). PoKs are interactive protocols in which a prover demonstrates knowledge of some kind to a verifier. PoKs play an important role in many cryptographic constructions and have found widespread applications in e-voting [25], [26], [28], [46], encryption [55], [56], group signatures [13], and private cryptocurrency transactions [20], [59].

More formally, a PoK involves two players: a *prover* $\mathcal{P}$ and a *verifier* $\mathcal{V}$. The goal is for $\mathcal{P}$ to convince an honest $\mathcal{V}$ that it knows a valid *witness* sk for some (public) statement x. In practice, PoKs are often *zero-knowledge* [33], meaning that the protocol hides any information about sk from $\mathcal{V}$. (A PoK need not be zero-knowledge, though, and this is true of some CK schemes we propose.)

As observed above, however, $\mathcal{P}$ could have access to the secret sk intermediated by an interactive protocol with another entity or device (e.g., a TEE or an MPC committee). In this case, classical PoK formalism breaks down, because it lacks a notion of encumbered knowledge.

In our voting example above, for instance, Alice has a secret key sk encumbered in a TEE that only allows her to cast a vote for Bob in an election. This same TEE, however, might allow unencumbered use of sk for completely different purposes, e.g., signing to authorize cryptocurrency transactions. In these contexts, Alice along with the TEE *can successfully prove knowledge* of sk. Yet *neither Alice nor any other entity truly knows* sk, in the sense of being able to use it for *any* desired purpose. In fact, the TEE can allow Alice to take any action using sk *except for the one to vote against Bob*. This allows a powerful collusion between Alice and the bribing adversary Mallory; Mallory can bribe Alice without risk since she provably cannot vote against Bob, while Alice is willing to accept the bribe since Mallory learns no information about the key and at the same time Alice is guaranteed to be able to utilize her key for all other functionalities.

This situation underscores a mismatch between the existing formalism for proofs of knowledge and knowledge in a critical, real-world sense. In this paper, we show the potential practical impact of this mismatch, describing coercive attacks that exploit encumbrance of secret keys in voting schemes, deniable messaging, and blockchain systems.

We show how to remedy the resulting problems by introducing the notion of CK.

Proofs of complete knowledge (PoCKs). In this work, we introduce and formalize proofs of complete knowledge (PoCKs). (Our definitions also cover *arguments* of complete knowledge (ACKs), for which $\mathcal{P}$ is polynomial time. We often informally use the term PoCK to denote both.)

The core idea in our formalization is intuitively as follows: A PoCK scheme ensures complete knowledge if it is the case that when an honest $\mathcal{V}$ accepts a proof by $\mathcal{P}$, $\mathcal{P}$ *can learn her own witness* sk *fully during the proof execution*.

Specifically, in a PoCK, $\mathcal{P}$ must be able to *eavesdrop* on an unencrypted channel carrying sk. To ensure this "self-eavesdropping" capability, $\mathcal{P}$ is given access to a special *resource* $\mathcal{R}$ required for successful execution of the proof.

$\mathcal{R}$ will typically be a local piece of hardware within the trust domain of $\mathcal{P}$. $\mathcal{R}$ may paradoxically itself be a TEE that stores sk—as a way of preventing encumbrance using another TEE. Alternatively, $\mathcal{R}$ could be a resource, such as an ASIC, with special computational capabilities. Informally, an *eavesdropper* $\mathcal{E}$ must be present on the channel between $\mathcal{P}$ and $\mathcal{R}$. Abstractly, the eavesdropper $\mathcal{E}$ can be visualized as the *physical manifestation* of a straight-line extractor [35] from PoK literature, in the sense that it will allow for extraction in practice rather than just as a proof construct (see Section IV for details).

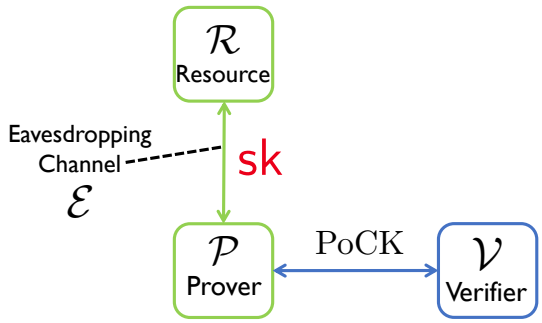

Fig. 1: *Proof of complete knowledge* (PoCK) setting. A PoCK is a proof of knowledge, but has two additional requirements: (1) $\mathcal{P}$ can only execute the proof successfully by accessing a special local *resource* $\mathcal{R}$; and (2) $\mathcal{P}$ must have access to the witness sk as it is transmitted to $\mathcal{R}$ over a plaintext *eavesdropping channel* $\mathcal{E}$. Green lines / boxes in the figure indicate entities local to / within the trust domain of $\mathcal{P}$, while blue lines / boxes indicate those outside.

In the case of $\mathcal{R}$ being a TEE, $\mathcal{E}$ can be constructed by simply requiring $\mathcal{P}$ to submit sk in plaintext to $\mathcal{R}$, or alternatively by having a function within the TEE application that reveals sk. Even if $\mathcal{P}$ uses a TEE or MPC committee in an attempt to encumber sk—thus hiding sk from herself—exposure on $\mathcal{E}$ means that $\mathcal{P}$ can still recover sk. Thus a PoCK ensures that sk is unencumbered.

Let's return to our voting example now. Intuitively, if a PoCK is used here, then Mallory will no longer have any guarantee on Alice's vote since Alice can use $\mathcal{E}$ to fully recover sk without being detected by Mallory.

The setting for a PoCK is shown in Figure 1.

Realizing CK. We consider two practical schemes for realizing PoCKs. These schemes enable any proof of knowledge protocol to be converted into a PoCK protocol.

Our first scheme is a conceptually straightforward one that generalizes an idea proposed by Gunn et al. [38] for preventing TEE-based attacks on deniability in messaging protocols. This scheme paradoxically realizes the resource $\mathcal{R}$ as a TEE *that prevents encumbrance* by, e.g., another TEE. The idea is that $\mathcal{P}$ generates sk inside or inputs sk to a special TEE application that outputs sk to the user on demand, thus realizing an eavesdropping channel $\mathcal{E}$. To prove complete knowledge of sk, $\mathcal{P}$ has the special TEE application generate an attestation proving that sk is output to $\mathcal{P}$. We prove the security of this CK scheme in the Universal Composability (UC) framework under the assumption that it is not practical to run a TEE instance inside another TEE instance.

TEEs have important drawbacks, though. Notably, one widely available, fully-featured hardware TEE is Intel SGX,

in which a number of serious vulnerabilities have been discovered, e.g., [18], [62], [63].[1]

For these reasons, we explore a second PoCK realization that involves *proof of work* (PoW) [32], [39], using an ASIC as the special resource $\mathcal{R}$.

An ASIC (Application-Specific Integrated Circuit) is special-purpose hardware for a specific computation. ASICs are widely used for cryptocurrency mining [60], specifically hashing (e.g., double SHA-256 for Bitcoin) and thus widely available as an off-the-shelf component for our PoCK scheme. ASICs also have an important feature for our purposes: they accept only *unencrypted* inputs.[2]

Our ASIC-based PoCK requires $\mathcal{P}$ to solve a specially crafted PoW puzzle within a certain time period. Successfully solving this puzzle will reveal sk to $\mathcal{P}$ with high probability. Complete description of our scheme along with the security analysis can be found in Section VI.

Interestingly, our approach to CK not only contributes to the theory of zero-knowledge proofs, but draws on techniques from the literature on this theory, specifically the *straight-line, non-programming extractor* construction of Fischlin [35].

To show the practicality of our CK schemes, we implement an end-to-end CK system SMACK (Section VII) in which the verifier is a smart contract on the Ethereum blockchain. This implementation both shows how minimal a CK verifier can be and supports the blockchain applications we discuss in Section II. The ASIC-based implementation (Section VII-C) uses an off-the-shelf mining ASIC. In order to make CK widely accessible, we also implemented our TEE-based scheme as a mobile app (Section B-A) using the mobile phone's TEE.

Contributions. In brief, our contributions are:

- *Complete Knowledge:* We introduce a new notion of knowledge for cryptographic protocols, called *complete knowledge* (CK), that addresses theoretical and practical limitations of classical proofs of knowledge arising from encumbrance of secrets.
- *CK applications:* We revisit known attacks on existing protocols that lack CK and offer a unified treatment and discussion of countermeasures through the lens of CK (Section II). We also introduce two new applications of CK that help prove ownership of blockchain assets.
- *Formalization:* We formalize CK as a strengthening of standard proofs / arguments of knowledge (Section III and IV).
- *Practical CK schemes:* We present two practical schemes to convert proofs of knowledge to proofs of complete knowledge (PoCKs), using a TEE (Section V) or off-the-shelf mining ASICs (Section VI). Our schemes work for a very broad class of Σ-protocols, a common type of ZK proof of knowledge in practice. *We implement both our CK schemes in a full, end-to-end system in which verification is performed in a smart contract (Section VII).*

[1]While a break of SGX or other TEEs may seem to *help* achieve CK by exposing secret sk, it can in fact *undermine* CK proofs by enabling the generation of fake attestations / proofs. At the same time, a TEE break does not necessarily prevent encumbrance of secrets, as the adversary that has broken a TEE may be distinct from one who is using the TEE to encumber secrets. Also, there are alternative ways to encumber secrets, e.g., MPC.

[2]Even if an ASIC supported encrypted connection, without a built-in enclave, it would still be eavesdroppable.

II. Motivation: Attacks and Applications

To motivate our exploration of CK, we now briefly review three key-encumbrance attacks from existing literature and explain how CK can serve as a countermeasure. The result is a new, unified treatment of these attacks that identifies their common root cause as a lack of CK for private credentials. We also describe two *new applications* enabled by CK; both relate to blockchains.

A. Attacks and CK-Based Countermeasures

Authentication protocols that lack CK are vulnerable to various attacks in which credentials, such as secret keys or passwords, are made available fully or conditionally to an adversary. Three concrete examples are described below.

Deniable Messaging: Deniability in messaging protocols is the (desirable) property that participants cannot prove the authenticity of a transcript of their communications to an outsider, rendering their communications inadmissible as evidence against them. Several widely deployed messaging protocols, e.g., Signal [1], [29] (which is based on OTR [17]), advertise such deniability as a key feature. These protocols accomplish deniability by exposing key material to two communicating parties, Alice and Bob, that allows either of them to forge a transcript unilaterally. E.g., Alice can do so using her long-term private key A and an ephemeral private key a.

Gunn et al. [38] show how a TEE can erode the deniable authentication [17], [31] that underpins deniable messaging protocols. The use of a TEE enables a simple attack on deniability by *either* communicating party in isolation. It suffices for Alice, for example, to generate her ephemeral private key a in a special TEE application that does not permit her to forge messages from Bob. Through this technique, Alice can show a judge that she lacks the ability to forge making it so that Bob loses deniability for the messages he sends.

Gunn et al. describe some countermeasures involving use of TEEs, the simplest involving use of a small TEE program that attests that a user's private keys are present in unprotected memory, i.e., *outside* the enclave, and thus made available to the user for transcript forgery.

Our work, by introducing CK, formalizes and extends ideas which allow the countermeasure from [38] to work.

Electronic Voting: Electronic voting is becoming important in decentralized systems such as permissionless blockchains [42], [46]. Those systems make use of user-generated keys. They are vulnerable to attacks—like those initially described in [30] and subsequently in [54]—in which a voter encumbers her private key used for voting in a TEE upon generation. The voter then offers a briber or coercer *exclusive and verifiable access* to her key either permanently or for certain elections. Access is verifiable because the TEE

can present a proof of encumbrance to the adversary. Note that the key can be encumbered for specific well defined tasks and does not for example, compromise the voter's cryptocurrency if the same key also controls her cryptocurrency.[3]

CK offers a countermeasure to such attacks. If a voter is required to prove CK for her private key sk, then she cannot encumber it.[4]

Similarly, CK offers a path to shoring up security in *minimal anti-collusion infrastructure* (MACI), a scheme proposed by Buterin [22] that is designed to provide bribery-resistance in voting and other applications in a way loosely analogous to coercion-resistance [14], [28], [41], [45].[5] Recent strides have been made toward MACI deployment on Ethereum [40]. As explicitly noted in [22], MACI is vulnerable to TEE-based encumbrance of keys. Use of CK, however, can restore the scheme's bribery-resistance properties.

B. New Applications Enabled by CK

We introduce two new applications enabled by the use of CK, i.e., through inclusion of PoCKs. These protocols show how CK can enable users to prove facts about asset ownership that they could not easily prove without the use of CK. Specifically, *these applications cannot be securely realized with standard PoKs*.

Key-coupling: In some systems, it is valuable to be able to ascertain that two different private keys, sk_1 and sk_2 (with respective public keys pk_1 and pk_2), are known simultaneously by the same user. This is possible with a CK witness consisting of two private keys. That is, the prover furnishes a CK proof of knowledge of $\mathsf{sk}_1 \parallel \mathsf{sk}_2$, i.e., a concatenation of the two private keys. We refer to such proof as *key-coupling*.

We emphasize that key-coupling is not possible using, e.g., a conventional PoK of $\mathsf{sk}_1 \parallel \mathsf{sk}_2$. Two distinct holders of sk_1 and sk_2 could jointly generate such a proof using secure function evaluation and avoid mutual key disclosure.

Key-coupling has a number of applications, particularly in blockchain systems. They include:

1) *KYC dilligence:* Cryptocurrency users often transfer control of their own coins from one key to another, e.g., from hot to cold wallets or vice versa. Users often must undergo know-your-customer (KYC) to transact with exchanges. It can be helpful for a user who has undergone KYC diligence with respect to the address associated with pk_1 to be able to transfer assets to an address associated with pk_2 *without having to undergo diligence again*. By proving simultaneous knowledge of sk_1 and sk_2, the user provides strong evidence that assets transferred between pk_1 and pk_2 belong to the same user—or at least fall under the control of a single entity. The same approach can be used by the owner of the address associated with pk_2 to prove that *she doesn't owe tax on funds* sent from pk_1, as the funds didn't change hands.

2) *Privacy-preserving credential linkage:* Suppose that a user who controls private keys associated with pk_1 and pk_2 has a public credential attached to pk_2 (e.g., proof of KYC diligence, as above). The user can construct a CK proof of knowledge of sk_1 *plus* possession of some public key / address (pk_2) with an associated KYC credential. She can do this *without revealing* pk_2.

3) *Enforcing NFT royalty payments:* Non-fungible tokens (NFTs) are blockchain objects that often represent ownership of digital artistic works. Some NFT platforms enforce *royalty payments* to artists (e.g., 5% of sale price) upon resale of an NFT. However, those platforms also support direct, royalty-free transfer between addresses so as to support transfer between addresses belonging to a single user. As there is no way (prior to our work) to determine single-owner possession of two distinct addresses, users can exploit this royalty-free transfer feature to *bypass* royalty payments. Controversially, for example, a popular marketplace called Sudoswap facilitates royalty-free NFT sales [34]. Key-coupling can, however, enforce *true single-owner possession of addresses in royalty-free transfers*, thereby closing the loophole that deprives NFT creators of ongoing royalty payments.

It is worth noting though that the ability of key-coupling to distinguish between within-owner and between-owner transfers might not be future-proof: User wallets could eventually support key changes, whether for key rotation or social recovery [19], [21]. This feature could be abused to transfer control between two distinct users while maintaining an appearance of consistent ownership by a single user. Still, the friction against cheating created by key-coupling might still be sufficient to protect, e.g., small-royalty NFT transfers.

CK addresses and Atomic NFTs: Blockchains enable new mechanisms for joint ownership of indivisible digital assets. Such ownership regimes are referred to as *fractionalization*. Fractionalization is a particularly popular approach to distributing ownership of expensive NFTs among a collection or syndicate of users.

Fractionalization, however, introduces problems such as price volatility and attractiveness to scammers [61]. As NFTs are essentially financial instruments—and usable not just for digital art, but for real-world assets such as real estate—preventing of fractionalization can also help with know-your-customer (KYC) / anti-money-laundering (AML) compliance, as it ensures that on-chain ownership representation is accu-

[3]Because a TEE can be taken offline, encumbrance in a TEE alone only ensures a briber that either the briber will be able to cast a vote or no one will. While limited, this property is still valuable to the adversary. Networks of TEEs, e.g., [6], [30] can in fact further help ensure liveness.

[4]*Coercion-resistant* voting protocols [14], [28], [41], [45] are another approach that aims to prevent adversarial interference. Such schemes involve either: (1) An authority that sends keys to voters over an untappable channel, along with the ability of users to present fake versions of these keys to adversaries or (2) The ability for the voter to re-vote. Unfortunately, a TEE can break coercion-resistance in either approach. CK *cannot* fully remedy the problem for approach (1), as the TEE can still identify true keys to adversaries. But CK *can* at least prevent an adversary from gaining *exclusive* access to voting keys and *can* restore the coercion-resistance for approach (2).

[5]The basic idea is to allow users to switch their registered keys secretly, thereby preventing an adversary from knowing whether a key presented by a user is valid or not. The scheme allows for a race condition between valid users and adversaries with whom keys are shared. It thus does not strictly meet formal definitions of coercion-resistance, e.g., JCJ [41], although use of deposits acts as a practical disincentive to key sharing.

rate [64]. Finally, certain types of NFTs, e.g., *soulbound tokens* (SBTs) [65] are intended by design for exclusive ownership; fractionalization would undermine their utility.

Fractionalization may be prevented on chain, i.e., on the blockchain itself, by allowing ownership only from a user address and not a smart contract. But there exists no mechanism (prior to our work) to prevent *off-chain* fractionalization of NFTs by means of secret sharing or TEEs.

CK offers a novel way to prevent fractionalization of any kind through a concept we call *CK addresses*. A CK address is one whose secret key is guaranteeed to be unencumbered through the use of a PoCK protocol. Through CK addresses, we can create what we refer to as *Atomic NFTs*—NFTs managed by a smart contract in a way that only permits ownership by CK addresses. This ensures that at all times *only one entity controls the NFT*.

III. Preliminaries and Background

We start by introducing some basic formalism. We introduce some notation and background in this section, including the basic formalism for interactive proof systems [37], [51].

A. Interactive Proof Systems

Computational model for interactive proofs. We adopt the standard Interactive Turing Machine (ITM) model [37] for protocol execution. An interactive proof system is a pair $(\mathcal{P}, \mathcal{V})$ of ITMs that communicate with each other in rounds. $\mathcal{P}$ and $\mathcal{V}$ may be given auxiliary inputs z_1 and z_2 respectively, along with a common input x. $\mathcal{V}$ outputs a single-bit at the end of the execution. We use $\langle \mathcal{P}(x, z_1), \mathcal{V}(x, z_2) \rangle$ to denote the random variable for $\mathcal{V}$'s output and $\text{VIEW}_{\mathcal{V}}(\mathcal{P}(x, z_1), \mathcal{V}(x, z_2))$ to denote the random variable for $\mathcal{V}$'s view of the execution.

Interactive proofs of knowledge. Consider a language $L \in \text{NP}$ with witness relation R_L, i.e., $x \in L$ iff. $(x, w) \in R_L$ for a *witness* w. Informally, the goal of a proof of knowledge system is to have the verifier output 1 iff $x \in L$ and the prover "knows" some witness w for x. We say that $(\mathcal{P}, \mathcal{V})$ is an interactive proof of knowledge system (for R_L) if it is *complete* and a *proof of knowledge*.

1) *Completeness* means that the honest prover $\mathcal{P}$ can always convince the honest verifier $\mathcal{V}$ when $(x, w) \in R_L$. Concretely, for all $(x, w) \in R_L$, we have $\Pr[\langle \mathcal{P}(x, w), \mathcal{V}(x) \rangle = 1] > 1 - \text{negl}(\lambda)$ where λ is the security parameter.

2) A *proof of knowledge* is a protocol in which, if a (malicious) prover $\mathcal{P}^*$ convinces $\mathcal{V}$ that $x \in \mathcal{L}$ (i.e., $\mathcal{V}$ outputs 1), then the prover "knows" a witness w for x. This is formalized by requiring the existence of an extractor $\mathcal{E}$ that can extract the witness given the description of $\mathcal{P}^*$. More formally, we require that for all $\mathcal{P}^*$ and x, $\Pr[w \leftarrow \mathcal{E}^{\mathcal{P}^*}(x) : (x, w) \in R_L] + \text{negl}(\lambda) > \Pr[\langle \mathcal{P}^*(x), \mathcal{V}(x) \rangle = 1]$. If this property holds only for a computationally bounded (PPT) adversarial prover, $(\mathcal{P}, \mathcal{V})$ is called an interactive *argument* of knowledge system.

3) We say a proof of knowledge is *zero-knowledge* if, informally, the verifier learns nothing from a proof execution. Specifically, for any PPT verifier $\mathcal{V}'$, there exists a PPT machine $\mathcal{S}$ (called the *simulator*) such that for all $(x, w) \in R_L$, it holds that $\text{VIEW}_{\mathcal{V}'}(\langle \mathcal{P}(x, w), \mathcal{V}'(x) \rangle) \approx \mathcal{S}(x)$, where $\approx$ means computational indistinguishability.

For a more detailed introduction, we refer the reader to [51]. As stated before, in a PoK system, there is no guarantee that the prover actually has unencumbered access to the witness.

Σ-protocols. A popular form of zero-knowledge proofs of knowledge (ZKPoK) used in practice are Σ-protocols. They are a key building block in our ASIC-based construction.

A Σ-protocol is a three-move, interactive ZKPoK with the following structure:

1) $\mathcal{P}$ sends a message a (often called a *commitment*) to $\mathcal{V}$.
2) $\mathcal{V}$ sends a challenge $c \leftarrow\!\!\$\ \{0,1\}^\ell$ to $\mathcal{P}$.
3) $\mathcal{P}$ sends a response s to $\mathcal{V}$.

For a given x, $\mathcal{V}$ decides whether to accept $\mathcal{P}$'s proof based on the proof transcript, which consists of the triple (a, c, s).

Σ-protocols have a property called *special soundness*, whereby an extractor can efficiently compute a witness w from a pair of accepting transcripts (a, c, s) and (a, c', s') where $c \neq c'$. (They also have a property called *special honest-verifier zero-knowledge*, which means that a simulator given x and c can generate a transcript (a, c, s) distributed like that in a real execution without access to the witness.)

An additional property of many Σ-protocols is *quasi-soundness*, which means that no efficient prover can produce any two transcripts of the form $(a, c, s), (a, c, s')$, where $s \neq s'$, i.e., two different responses for the same commitment-challenge pair. Our construction applies only to quasi-sound Σ-protocols.[6] We assume such Σ-protocols throughout.

The extractor $\mathcal{E}$ for a proof-of-knowledge protocol may in general *rewind* the prover $\mathcal{P}$. In a Σ-protocol, the special-soundness property gives rise to a simple extractor construction. After the first move in the protocol, i.e., the commitment a by $\mathcal{P}$, $\mathcal{E}$ issues a challenge c, and obtains response s. $\mathcal{E}$ then rewinds $\mathcal{P}$ to the point just after the first move and issues a second challenge c', recovering a second response s'. The two transcripts allow extraction of w.

The best-known Σ-protocol—and a practical choice for proving knowledge of a discrete-log-based public key in ASIC-ZKPoCK—is the Schnorr protocol [58], which proves knowledge of a discrete log. Here, $x = g^w$ (where g is a published generator of some suitable group $\mathbb{G}$), and the goal is to prove knowledge of the exponent w.

Straight-line extraction. The need to rewind the prover $\mathcal{P}$ results in loose security reductions for various signature schemes, e.g., [53] and is incompatible with the Universal Composability (UC) framework, an approach to proving secure composition of protocols [23]. These issues with rewinding motivated the exploration of *straight-line extractors* (a.k.a. *online extractors*), which do not require rewinding.

[6] Given the ability to generate two transcripts (a, c, s) and (a, c, s') for $s \neq s'$, for instance, which is permissible in strongly-sound Σ-protocols [43], a prover can cheat in our protocol.

Online extractors may observe calls to a hash function H by $\mathcal{P}$, where H is modeled as a *random oracle* (RO), i.e., returns responses that are distributed uniformly at random. Of particular interest in our setting are *non-programming* extractors, which involve a strong model that assumes that the extractor cannot program the RO, i.e., determine its responses during extraction.

Fischlin [35] proposed a non-interactive zero-knowledge proof of knowledge scheme with a straight-line extractor whose techniques we adapt to our ASIC construction. In this construction the extractor has access to RO queries. The key idea is that $\mathcal{P}$ uses Σ-protocol transcripts as inputs to the RO in a proof of work (PoW). The scheme is parameterized in such a way that solving the PoW requires with high probability that $\mathcal{P}$ feeds a pair of valid Σ-protocol transcripts $(a, c, s), (a, c', s')$ with $c \neq c'$ to the RO. By observing these transcripts as RO inputs, an extractor $\mathcal{E}$ can extract the witness w.

To provide more detail, Fischlin's scheme builds on a Σ-protocol involving a prover / verifier pair $(\mathcal{P}_\Sigma, \mathcal{V}_\Sigma)$. Parameters include challenge domain cardinality k, proof-of-work difficulty b, global proof-of-work target S, and number of rounds of execution n. Specifically, $H : \{0,1\}^* \rightarrow \{0,1\}^b$, and a valid PoW solution is such that $H(z) = 0^b$ for an input z that includes a valid Σ-transcript (a, c, s). To specify the protocol concisely:

- Prover $\mathcal{P}$: On input (x, w), $\mathcal{P}$ does the following:
  - Runs the first step of n independent executions of $\mathcal{P}_\Sigma$ to obtain n commitments $\vec{\mathsf{com}} = (a_1, a_2, \ldots, a_n)$. Let $q_i = ([x, i]; \vec{\mathsf{com}})$.
  - For each given a_i, $\mathcal{P}$ completes an execution of $\mathcal{P}_\Sigma$ on challenges $c_j \in [0, k-1]$. Each execution of $\mathcal{P}_\Sigma$ yields a corresponding response s_j. $\mathcal{P}$ sets $\pi_i = (a_i, c_j, s_j)$ for $H((a_i, c_j, s_j) \parallel q_i) = 0^b$ if one exists; otherwise it sets $\pi_i = (a_i, c_j, s_j)$ for the minimal result, i.e., j for $\operatorname{argmin}_j H((a_i, c_j, s_j) \parallel q_i)$.
  - Sends $\vec{\mathsf{com}}$, x, $\vec{\pi} = \{\pi_i\}_{i=1}^n$ to the verifier.
- Verifier $\mathcal{V}$: On input $(x, \vec{\pi})$, $\mathcal{V}$ checks: (1) For all $i \in [1, n]$ that $\mathcal{V}_\Sigma$ accepts π_i; and (2) $\sum_{i=1}^n H(\pi_i \parallel q_i) < S$.

This construction ensures straight-line extraction as follows. In order to compute PoW results that $\mathcal{V}$ will accept, $\mathcal{P}$ must with high probability hash *at least two* Σ transcripts of the form (a_i, c_j, s_j) for some a_i. That means that the random oracle H will be called on a pair of inputs that include distinct pairs (a_i, c_j, s_j) and (a_i, c'_j, s'_j). From quasi-soundness, it follows that w can be extracted.

As will be seen later, while our ASIC ACK protocol draws on the technique of combining a Σ-protocol with a PoW, our setting differs considerably. In straight-line extractors, extraction is a theoretical capability used to prove knowledge of w. In our ASIC ACK, extraction is a practical capability used to show that $\mathcal{P}$ can access w. Additionally, in our ASIC ACK, rather than using a *cryptographic* resource in the form of an RO, $\mathcal{P}$ uses a *computational* resource in the form of an ASIC. The result is a protocol that differs somewhat from Fischlin's in terms of the form of oracle queries involved and the resulting security analysis.

Trusted Execution Environments (TEEs). A TEE runs applications with strong confidentiality and integrity protections. Some TEE platforms can issue a type of statement, known as an *attestation*, to untampered execution of a particular application, along with application outputs.

One popular TEE with attestation capabilities is Intel Software Guard eXtensions (SGX) [10], [47], [48]. Trust in the hardware—and in Intel, which authenticates attestation keys—means that only an SGX platform can generate a valid attestation, i.e., attestations are existentially unforgeable. We make use of formalism for SGX-like TEEs in the universal composability (UC) framework from [50]. TEEs are nearly universal today in mobile devices as well, but without built-in attestation capabilities. Google and Apple, however, generate attestations for devices in their ecosystems [11], [12].

We make use of both SGX (Section V) and mobile-device TEEs (Appendix B-A) in different CK variants.

IV. Proofs of Complete Knowledge

As we have explained, the standard proof-of-knowledge property does not guarantee that the prover actually has unencumbered access to the witness. This is because in order to recover the witness, the (knowledge) extractor gets oracle access to the *entire* prover, including parts that may be controlled by separate entities or even the adversary. The issue is that the prover is modeled as a single machine $\mathcal{P}$ even if in reality it is not owned by a single entity. For instance, if the witness w is secret-shared between two independent parties, $\mathcal{P}$ will correspond not to the individual machines, but to the combined system with both parties, and the extractor is given access to this system. Here, even though the system as a whole knows the witness and the extractor is able to recover it, intuitively, it is clear the neither party alone really has unencumbered access to the witness.

In this section, we now formalize *proofs of complete knowledge* (PoCKs), which ensure that a single party has full access to the witness.

A. Building Intuition

The basic idea behind PoCKs is to design protocols where the knowledge extractor $\mathcal{E}$ is actually runnable in practice, rather than simply a proof construct. We therefore begin with a careful analysis of what can make an extractor fail if practically run, and then use these insights to guide our PoCK formalism. As a first point, since we want $\mathcal{E}$ to be able to run in practice, it is obvious that $\mathcal{E}$ should not need to rewind $\mathcal{P}$, i.e., it must be *straight-line*. We will further restrict our attention to straight-line (i.e., non-rewinding) extractors that are also *non-programmable* (i.e., unable to program e.g., the random oracle; this is important since real hash functions used to instantiate the random oracle cannot be programmed). This style of knowledge extractor has been used previously in e.g., [35], [43], [49].

Recall that for standard PoK extraction in the random oracle model, the extractor $\mathcal{E}$ is given two quantities: (1) A transcript

of the interaction between the prover and the verifier; and (2) A list of queries (and corresponding responses) made by the prover to the random oracle. What does it mean exactly for $\mathcal{E}$ to be given these inputs in practice? Unfortunately, we find that both of these inputs are problematic to assume in practice (due to encumbrance by MPC or a TEE) which makes designing protocols for our setting particularly challenging.

At an initial glance, the first input seems obtainable—any entity present on the communication channel between the prover and the verifier can observe this transcript (even if the prover is composed of multiple entities and only one communicates with the verifier). This however implicitly assumes that at some point, the communication is not encrypted and can therefore be observed by $\mathcal{E}$. Such an assumption fails if $\mathcal{V}$ is also run inside a TEE, in which case, a prover TEE holding the witness w would be able to convince $\mathcal{V}$ without w being known in plaintext to any non-TEE entity. This observation has a surprising consequence: the trivial PoK of simply sending w to $\mathcal{V}$ cannot be a PoCK unless it can be enforced that $\mathcal{V}$ is not run in a TEE (see Remark 2).

The second input—the list of random oracle queries—is also challenging to enforce in light of TEE or MPC encumbrance. This is because a hash function, which will typically be used to instantiate the random oracle, can be computed easily in trusted hardware or MPC. In turn, providing the oracle queries to $\mathcal{E}$ in practice would effectively translate to breaking the trusted hardware or unravelling the MPC protocol to figure out the hash function inputs. Intuitively, the gap here arises from the fact the *physical* instantiation of the random oracle may not comply with the way the extractor functions in theory for the proof of knowledge to go through.

Key technique. To surmount these challenges, we must ensure that the extractor always obtains the inputs required for knowledge extraction in practice. To do so, we consider a physical *resource oracle* functionality $\mathcal{R}$ such that the prover's interaction with $\mathcal{R}$ leaks a witness to $\mathcal{E}$. We seek to deploy $\mathcal{R}$ so that no prover can perform a successful CK proof without $\mathcal{R}$; in a sense, the resource abstraction separates out the part of the protocol responsible for the proof of knowledge.

Concretely, we model $\mathcal{E}$ as a man-in-the-middle entity for $\mathcal{R}$ that can snoop on the queries made to $\mathcal{R}$. $\mathcal{E}$ will use these queries to extract the witness, thereby giving it (and whoever can see its output) *complete knowledge* of the witness (see Remark 1). To emphasize its physical presence and its non-rewinding nature, we refer to $\mathcal{E}$ as the *eavesdropping extractor*, or simply the eavesdropper.

Looking ahead, in our protocols, $\mathcal{R}$ models special hardware available to $\mathcal{P}$—either a global SGX functionality that attests to seeing the witness, or an untrusted ASIC whose computational speed is superior to that of a trusted environment that can potentially conceal the witness from $\mathcal{E}$. In a sense, the SGX instance represents a *physical manifestation* of a trusted third party that furnishes the witness w directly to $\mathcal{E}$; the ASIC similarly may be thought of as a *physical manifestation* of an RO accessible by $\mathcal{E}$.

Here, $\mathcal{E}$ can be thought of as the machine where $\mathcal{R}$ physically resides; as an example, for the SGX resource, $\mathcal{E}$ represents the host machine of the SGX which may simply be one of the entities within $\mathcal{P}$ or even a different external machine. A crucial point here is that by design, we use a resource $\mathcal{R}$ such that there is no practical resource $\mathcal{R}'$ which provides an identical functionality and allows for encrypted queries to be made directly to $\mathcal{R}'$: this enables $\mathcal{E}$ to view the plaintext queries made to the resource.

Remark 1 (Complete knowledge for *some* entity)**.** Notice that PoCK only guarantees that the eavesdropper $\mathcal{E}$ can extract w. If for some reason, the prover does not have access to the output of $\mathcal{E}$ in practice, then it may not be able to recover w.

As a consequence, PoCK protocols can only guarantee that *some* entity (specifically $\mathcal{E}$ and anyone who can see its output) has complete knowledge of w. For instance, if $\mathcal{R}$ is connected to a different outsourced machine $\mathcal{M}$ instead of $\mathcal{P}$, then $\mathcal{E}$ will correspond to $\mathcal{M}$ and be able to recover the witness while $\mathcal{P}$ might not. We note that this subtlety is not accounted for in standard PoK formalism since $\mathcal{E}$ does not represent a physical entity.

This property, however, is sufficient to deter the collusion and bribery attacks which motivate our work. For instance, a user will not willingly accept a bribe for her vote if it reveals her key (which also holds her money) to another entity.

B. Formal PoCK Security

We now formally define proofs-of-complete knowledge. We use λ throughout to denote the security parameter.

Basic setting. Similar to standard PoK formalism, we consider a prover $\mathcal{P}$ and a verifier $\mathcal{V}$ which are modeled as ITMs. Specific to our setting, we model a *resource oracle* $\mathcal{R}$ that can be queried by $\mathcal{P}$. We will often work in a *timed* setting where $\mathcal{P}$ must complete its proof within some time $T(\lambda)$. To correctly model the concrete computational speed of a resource $\mathcal{R}$, we associate with it a function $t_{\mathcal{R}}(\cdot)$ that defines the time taken by the resource to compute responses to its queries. Note that this time can be concretely faster than $\mathcal{P}$.

Resource formalism. Abstractly, a resource $\mathcal{R}$ is a randomized and stateful functionality $\mathcal{F}_{\mathcal{R}}$. $\mathcal{R}$ is initialized with an internal state $\mathsf{st}_{\mathsf{initial}} \leftarrow\!\!\$\ \mathcal{R}.\mathsf{Setup}(1^\lambda)$. Upon input inp from $\mathcal{P}$, $\mathcal{R}$ computes $\mathcal{F}_{\mathcal{R}}(\mathsf{st}, \mathsf{inp}) \rightarrow (\mathsf{st}', \mathsf{out})$ where st is the current state of $\mathcal{R}$, st' is the state after the computation, and out is the output returned to prover. We also model the time taken by $\mathcal{R}$ as the randomized function $t_{\mathcal{R}}(\mathsf{st}, \mathsf{inp})$. Note that $t_{\mathcal{R}}(\cdot)$ may be smaller than if the computation was done by $\mathcal{P}$ itself. We use the tuple $(\mathcal{R}.\mathsf{Setup}, \mathcal{F}_{\mathcal{R}}, t_{\mathcal{R}})$ to represent $\mathcal{R}$.

PoCK formalism. Formally, a T-timed PoCK (when T is unspecified, there are no additional timing constraints) for a language $L \in \mathrm{NP}$ with witness relation R_L, and a class $\mathfrak{R}$ of resources is a tuple $(\mathsf{Setup}, \mathcal{P}, \mathcal{V})$ where:

- $\mathsf{Setup}(1^\lambda) \rightarrow pp$ is the setup algorithm that generates the public parameters.

- $(\mathcal{P}, \mathcal{V})$ is an interactive proof system where $\mathcal{P}$ is given (pp, x, w) and $\mathcal{V}$ is given (pp, x). $\mathcal{V}$ outputs a single bit indicating whether the prover has complete knowledge of a valid witness for x. For non-interactive proofs, $\mathcal{P}$ will output a proof π which will be given to $\mathcal{V}$ to verify. For T-timed protocols, $\mathcal{P}$ will be required to run in time T.

We can also consider the standard relaxation of *arguments* (instead of proofs) of knowledge for which only PPT provers are considered. Our concrete protocols similarly allow for Arguments of Complete Knowledge (ACK).

Now, for PoCK security, we define in the subsequent paragraphs, two properties—completeness, and forced-revelation (or CK-soundness)—that are required to hold for all resource oracles $\mathcal{R} \in \mathfrak{R}$.

Completeness. The first PoCK property of *completeness* mirrors the analogous property for PoKs. Recall that completeness states that an honest prover who holds the witness can convince the verifier to output the success bit. The only difference now for PoCK completeness is that the prover is endowed with a resource oracle $\mathcal{R}$. Formally, PoCK completeness states that for all $\mathcal{R} \in \mathfrak{R}$, all public parameters *pp* and $(x, w) \in R_L$:

$$\Pr[\langle \mathcal{P}^{\mathcal{R}}(pp, x, w), \mathcal{V}(pp, x) \rangle = 1] > 1 - \text{negl}(\lambda).$$

Furthermore, $\mathcal{P}^{\mathcal{R}}$ runs in time at most $T(\lambda)$ except with negligible probability.

Forced revelation. The second PoCK property of *forced revelation* is similar in spirit to the knowledge-soundness property of standard PoKs. Abstractly, if a prover is able to convince an honest verifier, then the eavesdropper will be able to output a valid witness. Let $\texttt{out}_{\mathcal{E}}$ denote the output of $\mathcal{E}$. A party able to view the output of the eavesdropper obtains the full witness. We also call this property CK-soundness. Formally, for all *pp*, inputs x, provers $\mathcal{A}$, and resources $\mathcal{R} \in \mathfrak{R}$ such that $\mathcal{A}^{\mathcal{R}}$ runs in time $T(\lambda)$,

$$\begin{aligned}\Pr[\langle \mathcal{A}^{\mathcal{R}}(pp, x), \mathcal{V}(pp, x) \rangle = 1] \\ < \Pr[x \in L \ \wedge \ (x, w = \texttt{out}_{\mathcal{E}}) \in R_L] + \text{negl}(\lambda).\end{aligned}$$

Forced revelation directly implies a couple of nice properties. First, it implies the usual soundness notion since no prover can convince $\mathcal{V}$ of an $x \notin L$ (except with negligible probability). More importantly, it also implies that if the prover does not make use of $\mathcal{R}$, making it so that $\mathcal{E}$ cannot eavesdrop, then it cannot convince $\mathcal{V}$ except with negligible probability *even if it has the witness*. Intuitively, this property is necessary because otherwise it would imply the ability to prove CK through e.g., a 2PC protocol where the witness is encumbered. As illustrated in the remark that follows, forced revelation also has surprising ramifications, which underscore the nuances of working in our PoCK setting.

Remark 2 (Trivial PoK and PoCK protocols)**.** Forced revelation implies an interesting separation between trivial protocols for PoK and PoCK. Recall that a trivial PoK protocol is for $\mathcal{P}$ to simply provide the witness to $\mathcal{V}$. Of course, such a protocol does not offer any privacy (e.g., zero-knowledge) properties. Notice however, that this would not be a PoCK protocol (since $\mathcal{E}$ does not come into play). This may seem surprising but is in fact an important consequence of the PoCK setting.

Abstractly, if the verifier receives the witness through an encrypted channel (e.g., through TLS), then it would also be possible for two parties constituting the prover and holding only shares of w to directly compute the required encryption of w for the channel. In such a case, the trivial PoK protocol could be simulated by two prover parties, neither of which has complete knowledge of w. Importantly, this is also possible if $\mathcal{V}$ is run inside a TEE since there is no point at which w is revealed in plaintext. Consequently, simply sending w will not be a PoCK protocol.

Intuitively, if we can guarantee that $\mathcal{V}$ is not run inside a TEE, then it will be possible to eavesdrop on the witness, making the trivial protocol a PoCK. One way this can be done is by having $\mathcal{V}$ itself be a TEE instance with the assumption that it is not practical to run a TEE inside another TEE.

This unearths a dependence of PoCKs on how the communication to $\mathcal{V}$ is defined, which is not seen for standard PoKs.

C. Zero-Knowledge PoCK

Most applications require that the witness held by the prover is not leaked to the verifier. A strong property typically considered in the PoK realm is that of *zero-knowledge* [37]. Informally, zero-knowledge ensures that no additional information is leaked to the verifier. We will add a similar requirement to PoCKs to formalize "Zero-Knowledge Proofs of Complete Knowledge" or ZKPoCKs.

Zero-Knowledge property for PoCKs. We now adapt the standard zero-knowledge definition to our setting. Formally, we define the zero-knowledge property as follows: For any PPT verifier $\mathcal{V}'$, there exists a PPT machine $\mathcal{S}$ (called the simulator) such that for all $\mathcal{R} \in \mathfrak{R}$, $(x, w) \in R_L$ and auxiliary input $z \in \{0,1\}^*$, it holds that:

$$\text{VIEW}_{\mathcal{V}'}(\mathcal{P}^{\mathcal{R}}(pp, x, w), \mathcal{V}'(pp, x, z)) \approx \mathcal{S}(pp, x, z).$$

D. Eavesdropper Undetectability

Our current PoCK formalism models $\mathcal{E}$ as a concrete man-in-the-middle entity which eavesdrops on queries made to the resource $\mathcal{R}$. We implicitly assume that the $\mathcal{E}$ is *always run*; in other words, we do not model a scenario where the witness was not extracted even though it *could have been*. While this distinction is not important in the standard cryptographic context, and therefore not part of our core PoCK formalism, as we will see, it uncovers subtleties in the context of side channels and incentive compatibility. The purpose of this section is to bring to light these subtleties and extend our formalism to accommodate for them.

Side channel on the usage of $\mathcal{E}$. To differentiate between whether $\mathcal{E}$ was used to recover the witness or not, we consider a side channel that provides a remove adversary $\mathcal{A}$ with this information. Consequently, $\mathcal{A}$ can now take actions based on whether the witness was extracted; notably, as we show later, this gives $\mathcal{A}$ additional advantages in our vote bribery scenario.

It is important to emphasize here that the witness can always be extracted, (thereby satisfying CK-soundness); the difference is in *whether it actually was*. We also note that this detection ability is a highly unconventional power given to the adversary; while $\mathcal{A}$ cannot itself recover the witness, it is still made aware of whether $\mathcal{P}$ did. This is non-standard in the context of existing literature—when $\mathcal{R}$ (and $\mathcal{E}$) is not in the domain of $\mathcal{A}$ (because otherwise $\mathcal{A}$ could extract the witness itself), we note that there is likely no practical side channel that reveals to $\mathcal{A}$ whether $\mathcal{P}$ chose to extract.

Still, to ensure that PoCK protocols can be correctly deployed in applications where side channels need to be accounted for, we introduce an explicit assumption—*eavesdropper undetectability*—as the property that an adversary cannot detect whether $\mathcal{E}$ was run or not. We show how this property is critical for incentive-compatibility in our vote bribery example. This also serves to highlight the non-triviality of our CK setting.

Formal description. To define eavesdopper undectability, we relax our earlier modeling assumption that $\mathcal{E}$ exists as a man-in-the-middle entity for $\mathcal{R}$ and can snoop on any queries made to it. Instead, we will give the prover the ability to make queries to $\mathcal{R}$ *without the usage of* $\mathcal{E}$; we use $\mathcal{R} \setminus \mathcal{E}$ to denote this oracle. The honest prover $\mathcal{P}$ will still make use of $\mathcal{E}$.

We can now define eavesdopper undectability as the following property: For pp, inputs x, and $\mathcal{R} \in \mathfrak{R}$, for all (possibly malicious) provers $\mathcal{P}'$ and verifiers $\mathcal{V}'$, there exists $\widehat{\mathcal{P}}$ such that the following ensembles are indistinguishable:

$$\begin{aligned} &\text{VIEW}_{\mathcal{V}'}\left(\mathcal{P}'^{\mathcal{R}\setminus\mathcal{E}}(pp,x),\mathcal{V}'(pp,x)\right) \\ \approx\ &\text{VIEW}_{\mathcal{V}'}\left(\widehat{\mathcal{P}}^{\mathcal{R}}(pp,x),\mathcal{V}'(pp,x)\right). \end{aligned}$$

Intuitively, this means that no $\mathcal{V}'$ can distinguish whether it is interacting with a prover which uses $\mathcal{E}$ or one which does not.

In the example that follows, we briefly describe how a side channel which informs $\mathcal{A}$ of whether $\mathcal{E}$ was run breaks security in our vote bribery scenario. Eavesdropper undetectability is required here to prevent this attack. We leave further exploration of this property to future work.

Incentive compatibility example. Suppose that a PoCK protocol Π is used in a voting application to mitigate the risk of key-encumbrance based bribery. As stated earleier, Π ensures that the eavesdropper $\mathcal{E}$ *can recover* the secret key but says nothing about *whether the recovery is actually carried out*—our previous CK formalism implicitly assumes that $\mathcal{E}$ will always output the recovered key.

Still, if there was some side channel through which a remote adversary $\mathcal{A}$ could detect whether the key was extracted (by $\mathcal{E}$ or $\mathcal{P}$ in general), then it could condition its bribe on this extraction action not being taken. The consequence of such a conditional bribe is that while $\mathcal{P}$ has the ability to learn her key, she will be *incentivized not to do so* in order to profit from the bribe. In particular, while $\mathcal{P}$ *can* always learn her full key, making the protocol satisfy CK-soundness, $\mathcal{A}$ will be able to detect such an action and refuse to pay $\mathcal{P}$; this incentivizes $\mathcal{P}$ to not learn her key even if she is able to. The key will therefore remain encumbered. Eavesdropper undetectability removes this side channel vulnerability.

V. SGX-based PoCK Protocol

We now describe SGX-PoCK, a simple but illustrative PoCK protocol which uses an SGX TEE as its resource. Intuitively, for this PoCK, the SGX models a physical manifestation of a trusted third party to whom the prover will submit the witness. We use SGX for concreteness but note that a similar PoCK can be realized through any TEE which admits remote attestation, including those in mobile devices, as discussed in Section B-A.

SGX resource. We model the SGX resource by using the formalism for TEEs with attested execution from Pass et al. [50]. Abstractly, SGX attestation is modeled using a global functionality (i.e., one that allows for global setup) $\mathcal{G}_{\mathsf{SGX}}$ within the GUC-framework [24]. $\mathcal{G}_{\mathsf{SGX}}$ models all valid SGX processors and is initialized with a master key pair $(\mathsf{mpk}, \mathsf{msk})$ with signature scheme $\mathrm{S} = (\mathrm{S.kg}, \mathrm{S.sign}, \mathrm{S.ver})$; this intuitively allows the modeling of anonymous attestation which prevents identifying the SGX which signed an attestation.

$\mathcal{G}_{\mathsf{SGX}}$ permits SGX-equipped parties (denoted by the set Reg) to install programs on their SGX and compute outputs. When a party X provides input inp to an installed program prog, $\mathcal{G}_{\mathsf{SGX}}$ computes its output out and a signature σ on $(\mathsf{id}, \mathsf{prog}, \mathsf{out})$ where id denotes UC-relevant session identification information. The tuple (out, σ) is then sent to the querying party X as the attested output. For completeness, we detail the full $\mathcal{G}_{\mathsf{SGX}}$ functionality in Fig. 8 in Appendix A.

SGX-PoCK description. We describe the full SGX-PoCK protocol in Fig. 2. Abstractly, the prover $\mathcal{P}$ first installs the program $\mathsf{prog}_{\mathsf{CK}}$ through $\mathcal{G}_{\mathsf{SGX}}$. Now, given (x, w) in the relation R_L as input, $\mathcal{P}$ submits the tuple ("expose", x, w) to $\mathcal{G}_{\mathsf{SGX}}$ and gets back a signature σ on $(\mathsf{id}, \mathsf{prog}_{\mathsf{CK}}, (\text{"exposed"}, x))$ which it forwards to $\mathcal{V}$. By checking the validity of the signature, $\mathcal{V}$ can convince itself of complete knowledge of a witness corresponding to x.

A. SGX-PoCK Properties

It is easy to see that an honest $\mathcal{P}$ given w can always convince $\mathcal{V}$; in other words, completeness holds for SGX-PoCK. For CK-soundness, we require an assumption on the infeasibility of specific types of resources, as we describe below:

Resource assumptions. To ensure that the witness exposed to the SGX can be eavesdropped upon, intuitively we need to assume that it is not practical to run $\mathsf{prog}_{\mathsf{CK}}$ in an SGX within another SGX. This is because otherwise, the outer SGX could be in possession of the witness w which it could expose to the inner SGX to obtain a CK proof without w ever being accessible outside of a trusted enclave. Remark 3 briefly examines how this assumption can be removed.

In the context of the $\mathcal{G}_{\mathsf{SGX}}$ formalism, this means that no program prog installed by a party can install its own program prog'; note that this is implicitly assumed within [50] since only a fixed registration set Reg is considered for SGX devices.

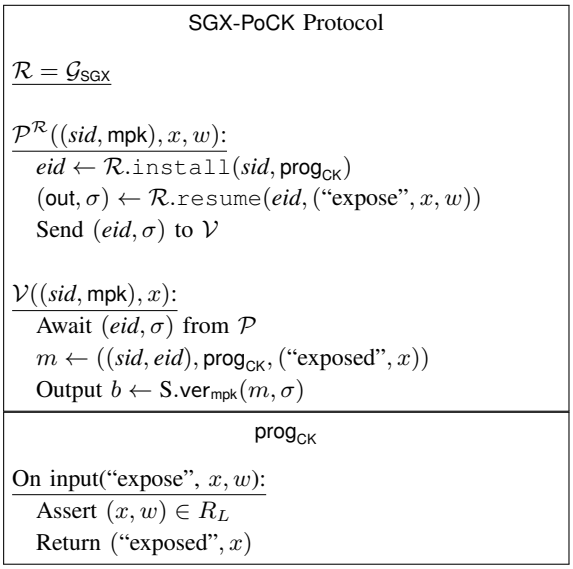

Fig. 2: SGX-PoCK Protocol Description.

CK-soundness proof. Now, assuming that there is no practical resource $\mathcal{R}'$ that models such a 2-layer SGX, it is straightforward to show that SGX-PoCK satisfies CK-soundness.

Consider a prover $\mathcal{P}'$ that is able to convince the honest verifier $\mathcal{V}$ that it knows the witness to a statement x. This can happen in only one of two ways: (1) $\mathcal{P}'$ submits (x, w) to $\mathcal{R} = \mathcal{G}_{\mathsf{SGX}}$ as one of its queries; (2) $\mathcal{P}'$ does not query $\mathcal{R}$ with (x, w)—it either does not use $\mathcal{R}$ at all or queries it with different values. In the first case, the witness w will be sent in plaintext to $\mathcal{R}$ allowing, allowing $\mathcal{E}$ to easily output it. The second case arises only with $\mathrm{negl}(\lambda)$ probability given the SUF-CMA security of the signature scheme S used.

Remark 3 (SGX inside SGX)**.** SGX-PoCK can in fact be modified to work even when an SGX can be run inside another SGX as long as this can be done only a finite number of times. Specifically, if is practical to run k layers of SGX but not $k+1$ layers, then SGX-PoCK can be modified to use the k-layer SGX as the resource $\mathcal{R}$; $\mathcal{P}$ now obtains an attestation from this $\mathcal{R}$. By assumption, since a $(k+1)$-layer SGX is not practical, the witness submitted to $\mathcal{R}$ will be seen by $\mathcal{E}$ allowing for extraction.

This also serves to future proof our protocol in case of advances in TEE infrastructure.

Privacy properties. Observe that SGX-PoCK as described is not zero-knowledge since the attestation can be forwarded. Still, the SUF-CMA security of S directly implies non-trivial privacy properties over the basic PoCK. In particular, given many SGX-PoCK proofs (which are nothing but signatures using the master key-pair), an adversary still cannot forge a different SGX-PoCK proof for any other statement x.

Making SGX-PoCK satisfy zero-knowledge. Intuitively, to make the protocol zero-knowledge, there must exist a simulator $\mathcal{S}$ that can simulate the protocol transcript without access to the witness. Further note that $\mathcal{S}$ should also not be able to program the master secret key of the SGX (this is accounted for since we model the SGX resource as a GUC functionality).

We now briefly describe how SGX-PoCK can be made zero-knowledge. As is the case with other GUC protocols, $\mathcal{S}$ will require a trapdoor in other to simulate transcripts. Towards this, intuitively, we introduce a trapdoor τ which allows the generation of arbitrary SGX attestations for our protocol.

Specifically, $\mathcal{V}$ first chooses a trapdoor τ and submits it to $\mathcal{G}_{\mathsf{SGX}}$ which returns an attestation σ_c on $c = \mathsf{owf}(\tau)$ where owf is a one-way function. $\mathcal{V}$ sends the (c, σ_c) to $\mathcal{P}$ who then submits (c, x, w) to $\mathcal{G}_{\mathsf{SGX}}$. Before providing an attestation that the witness w was seen, $\mathcal{G}_{\mathsf{SGX}}$ ensures that the correct c was input. Finally, the $\mathcal{V}$ can check the correctness of the attestation to complete the proof. Note that this protocol also requires $\mathcal{V}$ to possess an SGX device.

This construction will now be zero-knowledge since the simulator $\mathcal{S}$ can use the trapdoor τ to forge attestations simulate the interaction between $\mathcal{P}$ and $\mathcal{V}$.

VI. ASIC-Based PoCK Construction

In this section, we explore the design of a (ZK)PoCK using a cryptocurrency-mining ASIC as the prover resource $\mathcal{R}$—a protocol we call ASIC-ZKPoCK. Our construction is quite general. It can transform a broad class of Σ-protocols [57]—a common class of three-move, honest-verifier, zero-knowledge proof of knowledge (ZKPoK)—into a ZKPoCK through the use of an ASIC. The only requirement is that the Σ-protocol be quasi-sound.

Intuition. ASIC-ZKPoCK makes use of the *performance gap* between computation in secure environments (e.g., SGX) or secure multi-party computation (MPC) and computation using fast ASIC hardware. By running as a time-constrained protocol, ASIC-ZKPoCK ensures that it is only feasible to compute a correct, timely proof using a mining ASIC.

As required, mining ASIC hardware has an eavesdropping channel $\mathcal{E}$. (Mining ASICs, as we explain below, don't support encryption, so eavesdropping is straightforward.) This channel $\mathcal{E}$ allows the prover to extract the witness during the proof generation process, ensuring complete knowledge.

In short, a mining ASIC may be viewed as a computing resource $\mathcal{R}$ that is special in that it is fast—faster than a CPU—and has an eavesdropping channel $\mathcal{E}$ on inputs. A mining ASIC thus fits our basic CK framework shown in Fig. 1.

Why ASIC-based (ZK)PoCKs? There are two reasons, security and performance related respectively, for exploring ASIC-based PoCKs over TEE-based PoCKs. First, many TEE-based machines operate in the cloud. As noted in Section I, TEE vulnerabilities could expose the private keys to cloud operators or remote adversaries with access to the cloud. Second, in the case of blockchain applications, TEE attestations can be expensive to verify. For example, EPID [3] attestations—an attestation type generated by Intel SGX without special provisioning and with optional privacy protection—is expensive to verify in the Ethereum Virtual Machine (EVM).

In the remainder of this section, we start with a brief background on cryptocurrency-mining ASICs (Section VI-A). We then present the detailed ASIC-ZKPoCK protocol (Section VI-B) and analyze its security (Section VI-C).

A. Background: Cryptocurrency-Mining ASICs and PoWs

Certain cryptocurrencies—predominantly Bitcoin today—use *proof of work* (PoW) [39] for the safety of their underlying consensus mechanism for block generation. PoW involves solving puzzles by means of repeated cryptographic hashing. (We give these puzzles' exact format below.) The process of using PoW to generate blocks is known as *mining*.

A mining ASIC is designed for fast PoW puzzle solving. It can compute hashes far faster than a CPU—typically by more than a factor of 1,000,000—and thus achieves a performance gap with respect to any SGX-protected application for hashing.

Mining ASICs today take only *unencrypted* inputs, meaning that their inputs are exposed to users.[7]

To provide some notation that we use in what follows, a PoW puzzle is based on a particular hash function $\mathcal{H}$ (typically modeled as a random oracle) with ℓ-bit outputs, where ℓ is the security parameter. A puzzle instance has a *difficulty* d corresponding to the probability of correctly solving it with a single hash computation.[8] A puzzle instance may also include ancillary data B. For PoW cryptocurrencies, B consists of header data from the block the miner is attempting to mine. Solving a puzzle with ancillary data B and difficulty $d \in [1, \infty)$ involves finding a *nonce* ν such that $\mathcal{H}(B \parallel \nu) < 2^\ell/d$. The probability of solving the puzzle for any random nonce is an independent and identically distributed Bernoulli random variable with success probability $1/d$ (for $d \mid 2^\ell$)

The idea behind ASIC-ZKPoCK is to require the prover $\mathcal{P}$ to use an ASIC to find a solution π to a proof-of-work puzzle like those in Bitcoin mining. The puzzle solution π is required to include a proof of knowledge of the witness w. Specifically, $\mathcal{P}$ specifies PoK commitment a and π includes the challenge c and response s for a transcript (a, c, s) of a Σ-protocol involving w. A Σ-protocol is zero-knowledge, meaning that the transcript—and thus the puzzle solution π—does not expose w.

However, the process of computing π involves $\mathcal{P}$ sending *multiple* Σ-protocol transcripts to the ASIC. Given the special soundness (and quasi-soundness) of the Σ-protocols we use, a prover accessing $\mathcal{E}$ *can with high probability extract* w.

It is possible in principle to perform mining for an ASIC-ZKPoCK in an SGX enclave (in a CPU), but not realistically in practice, as ASICs are far more performant than CPUs. As we explain below, a top-of-the-line ASIC may be expected to outperform a (single-server) SGX application by a factor of more than 1,000,000.

[7]Even if such ASICs were ultimately to support encryption, they also do not support enclaves, so we may presume any value input to an ASIC will still be exposed to the user. While one could imagine reasons to support encryption in mining ASICs, there's no compelling reason to support enclaves.

[8]We refer to d generically in our protocol description as a difficulty parameter, without reference to the specific notion of "difficulty" in Bitcoin.

B. ASIC-Based PoCK: Protocol construction

Formally, in ASIC-ZKPoCK, $\mathcal{P}$ and $\mathcal{V}$ execute a Σ-protocol. $\mathcal{P}$ embeds a valid proof transcript (a, c, s) for the Σ-protocol in a PoW puzzle whose solution π constitutes a full ASIC-ZKPoCK proof. The key idea in our construction is to *require $\mathcal{P}$ to try out multiple puzzles, each with a different challenge c (and thus response s), in order to find a solution*. We accomplish this by carefully choosing parameters such that with high (but still constant) probability, a single randomly chosen c will not lead to a puzzle with a valid solution. As a result, $\mathcal{P}$ must *input different transcripts to the ASIC* (or in other words, create different puzzles for the ASIC), among which, by quasi-soundness, is a pair $(a, c, s), (a, c', s')$ with $c \neq c'$ and $s \neq s'$. From this pair, given the special soundness property of the Σ-protocol, $\mathcal{E}$ can extract w.

At the same time, however, π itself—the solution revealed to the verifier—contains *only one proof transcript*. Thus $\mathcal{V}$ does not learn w ensuring that the protocol remains zero-knowledge.

As $\mathcal{P}$ must complete the proof in a limited period, it can succeed only with a powerful resource. Given the right choice of security parameter, this means that $\mathcal{P}$ must employ an ASIC. We will consider the set $\mathfrak{R}$ of resources that work to be all PoW ASICs with hash rate at least some threshold Q_{asic}.

Preliminaries. ASIC-ZKPoCK involves a random *PoCK challenge* r from $\mathcal{V}$. Let $\pi = (B, \nu)$ denote a puzzle solution computed by $\mathcal{P}$ in response to a PoCK challenge r. The puzzle solution π consists of a block header B and nonce ν corresponding to a valid proof of work. Let $\Sigma\mathsf{map}_r(\pi) \rightarrow (c, s)$ denote a function, dependent on r (as specified below), that maps π to a pair (c, s).

We define two verification functions:

- $\mathsf{PoKAccept}(x, (a, c, s)) \rightarrow \{\mathsf{true}, \mathsf{false}\}$ checks the correctness of the Σ-protocol transcript (a, c, s) with respect to public PoK value x.
- $\mathsf{puzAccept}[d, \beta](\pi) \rightarrow \{\mathsf{true}, \mathsf{false}\}$ checks the block / nonce pair (B, ν) represents a correct puzzle solution with difficulty d, i.e., $\mathcal{H}(B, \nu) < 2^\ell/d$. puzAccept also checks that nonce ν is of correct size, namely $\nu < \beta$, for a parameter β discussed below.

A PoW protocol gives a probabilistic estimate of the work done by a prover. For PoW schemes like ours with multiple puzzle solutions, it is possible to generalize away from verifying individual puzzle solutions to verifying a collection of puzzle solutions jointly. We therefore consider a variant of puzAccept, namely $\mathsf{puzAccept}[d, \beta](\pi_1 ... \pi^n))$, that checks puzzle solutions $(\pi_1, \pi_2, ... \pi_n)$ across n rounds of our ASIC-ZKPoCK protocol. We consider two concrete versions in our analysis later in this section: (1) $\mathsf{puzAccept}^{\mathsf{indiv}}[d](\pi_1 ... \pi_n)) = \bigcap_{i=1}^{n} \mathsf{puzAccept}[d, \beta](\pi_i)$ checks that each *individual* solution satisfies the difficulty level; (2) $\mathsf{puzAccept}^{\mathsf{agg}}[d](\pi_1 ... \pi_n))$ more accurately reflects aggregate work across all solutions by checking whether $\sum_{i=1}^{n} \mathcal{H}(B_i, \nu_i) < n2^\ell/d$.

Efficient puzzle-solving vs. transcript extraction. As explained above, we must design our ASIC-ZKPoCK protocol so

that it forces $\mathcal{P}$ to try different values of c while computing a puzzle solution π. Changing c, however, carries the overhead of computing a new a new corresponding block header B and feeding B to the ASIC. To ensure a tight bound on the proving time, we don't want $\mathcal{P}$ to have to change c *too* frequently. Both the variants also check that that nonce ν is of correct size, namely $\nu < \beta$, for a parameter β discussed below.

Our approach to resolving this tension is to: (1) map a given block header B to a distinct pair (c, s), so that changing B changes (c, s) *but* (2) allow $\mathcal{P}$ to explore a range of different nonce values ν for a given B.

We construct a mapping $\Sigma\mathsf{map}_r(\pi) \to (c, s)$ as follows. Recall that $\pi = (B, \nu)$. The function $\Sigma\mathsf{map}_r$ partitions $B \to B[1] \parallel B[2]$. It computes $c = \mathcal{H}(B[1] \parallel r)$ from $B[1]$. We can view $\mathcal{H}(\cdot \parallel r)$ here as a hash function selected at random by $\mathcal{V}$ by means of the PoW challenge r in order to prevent PoW precomputation.

Because s depends on c (and a), $\Sigma\mathsf{map}_r$ is constructed such that $\mathcal{P}$ can specify $s = B[2]$, i.e., encode s in a portion of B distinct from that for c. Given the collision-resistance of $\mathcal{H}$, changing $B[1]$ of course changes c. Changing $B[2]$ also changes $B[1]$: Given a fixed a, a given challenge c has only one corresponding correct response s, due to the quasi-soundness property discussed in Section III. In short, $\mathcal{P}$ cannot feasibly construct distinct blocks B that map to the same pair (c, s).

At the same time, we allow $\mathcal{P}$ to explore the space of possible nonces ν. To ensure that $\mathcal{P}$ is forced to try multiple puzzles (and thus multiple (c, s)), the nonce space should not be too large. Therefore, we impose in puzAccept the restriction $\nu < \beta$ for a protocol parameter β.

In summary, $\mathcal{P}$ can feed a block header B to an ASIC to solve a puzzle corresponding to some challenge c. Provided that the security parameter β is small enough—i.e., the space of valid ν is small—the probability of $\mathcal{P}$ finding a puzzle solution $\pi = (B, \nu)$ for any single value of c is low. Therefore, $\mathcal{P}$ must w.h.p. try multiple values of c to find a puzzle solution. Consequently, $\mathcal{P}$ is likely to use a pair of triples $(a, c, s) \neq (a, c', s')$ from which w can be extracted.

ASIC-ZKPoCK protocol. ASIC-ZKPoCK is presented in Fig. 3. Two points are worth highlighting. First, we reiterate that the protocol is *interactive*; $\mathcal{V}$ supplies a PoW challenge r as described above. Second, the protocol is *timed*; $\mathcal{P}$'s response is only accepted by $\mathcal{V}$ if it is returned within time τ. The goal, again, is to ensure that computation has taken place in a (fast) ASIC, rather than a (relatively slow) CPU.

C. Security Analysis

We now analyze the security properties satisfied by ASIC-ZKPoCK. In particular, we show that it satisfies the PoCK properties of completeness and forced revelation. For simplicity of illustration, we analyse the security of our scheme by instantiating $\mathsf{puzAccept}[d, \beta](\pi_1, ..., \pi_n)$ with $\mathsf{puzAccept}^{\mathsf{indiv}}$. Note that utilizing the alternative $\mathsf{puzAccept}^{\mathsf{agg}}$ only improves the security of our scheme. We further demonstrate the practicality of ASIC-ZKPoCK by choosing concretely viable parameters.

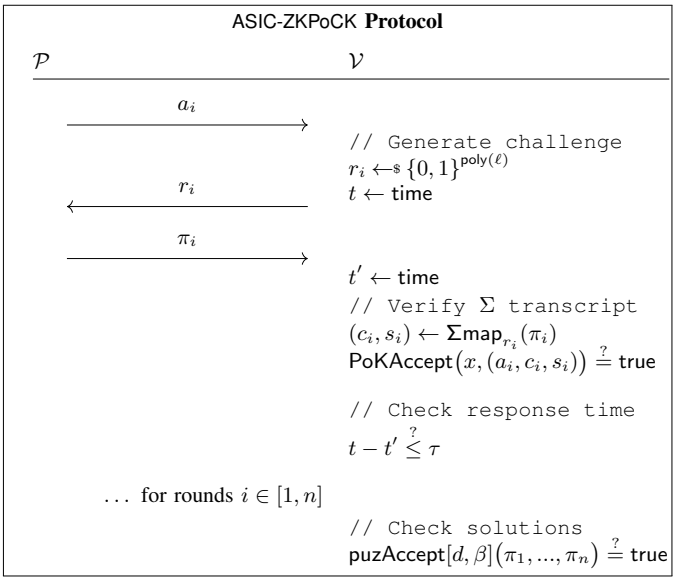

Fig. 3: ASIC-ZKPoCK protocol. The protocol executes over n rounds. $\mathcal{V}$ runs PoKAccept in each round to check that $\mathcal{P}$ has specified a valid Σ transcript (a_i, c_i, s_i) and also checks that $\mathcal{P}$ has run within time bound τ. After n rounds, $\mathcal{V}$ runs puzAccept to check that the joint set of PoW puzzle solutions $\pi_1, ..., \pi_n$ is correct and represents a quantity of work exceeding a lower bound specified by d.

B	Block header	d	PoW puzzle difficulty
ν	PoW puzzle nonce	τ	Prover time bound
π	PoW solution	β	Bound on nonce ν size
r	PoCK challenge	ℓ	Security parameter
(a, c, s)	Σ-protocol transcript	n	Number of rounds

Fig. 4: Protocol notation (left) and parameters (right)

We begin with a simple lemma that bounds the probability of failure to compute a valid puzzle solution.

Lemma 1. *Let p^Q_{fail} denote the probability of $\mathcal{P}$ with hash rate Q failing to compute a puzzle solution. Define $k = \tau Q/d$. Intuitively, k reflects the adversary's computational resource over the lifetime of a challenge relative to puzzle difficulty, i.e., higher k means more adversarial advantage. Then:*

$$e^{-k}\Big(1 - \frac{k}{d}\Big) \leq p^Q_{\mathsf{fail}} \leq e^{-k}. \tag{1}$$

Proof. In time τ, $\mathcal{P}$ can compute $Q\tau = kd$ hash evaluations. Recall that the probability of finding a solution for a single evaluation is $1/d$. Therefore, p^Q_{fail} can be given by $(1-1/d)^{kd}$. The bounds follow from simple exponential inequalities. □

1) Completeness

The completeness error of ASIC-ZKPoCK depends on the hashrate of the honest prover as shown below. It is easy to see that as the hashrate Q increases, the completeness error drops exponentially.

Observation 1. *For a prover $\mathcal{P}$ with hash-rate Q,* ASIC-ZKPoCK *satisfies completeness with error $\epsilon \leq ne^{-Q\tau/d}$.*

Proof. By the union bound, it holds that $\epsilon \leq np_{\mathsf{fail}}^{Q}$. Therefore, by Lemma 1, we can conclude $\epsilon \leq ne^{-Q\tau/d}$. □

2) Forced Revelation

In general, for ASIC-ZKPoCK to function correctly, it should be infeasible for an adversary to execute the protocol in an enclave (thus on a CPU). Unfortunately, mining puzzles are embarrassingly parallel, which means that in principle, an adversary can use a network of multiple TEE-enabled hosts to solve them. We must therefore characterize security in terms of the size of the network.

Furthermore, even assuming that an ASIC is used, in order for $\mathcal{E}$ to extract the witness, we also need to show that at least two distinct challenges c and c' are used in the computation.

Consequently, to show forced-revelation, we need to bound, for both strategies, the probability that the adversary wins.

Observation 2. *Define $u = d/\beta$, where β is the security parameter for the range of the nonce. The probability $p_{\mathsf{succ}}^{\mathsf{onechal}}$ of an adversary (irrespective of hash-rate) computing a valid puzzle solution π using a single challenge c in n rounds is:*

$$p_{\mathsf{succ}}^{\mathsf{onechal}} < \left(1-\left(1-\frac{1}{d}\right)^{\beta}\right)^{n} \leq \left(1-\left(1-\frac{1}{du}\right)e^{-1/u}\right)^{n}. \quad (2)$$

A secure parameter setting for a single round of ASIC-ZKPoCK will therefore require large enough u—and thus β such that $\beta \ll d$—so that $p_{\mathsf{succ}}^{\mathsf{onechal}}$ is small.

An adversary can alternatively seek to boost its mining rate while preventing disclosure of w by using a network of m distinct enclave-enabled CPUs for large m. While the adversary can also concurrently use a mining ASIC for one value of c, as noted above, a secure parameter setting for ASIC-ZKPoCK will involve an exponentially small $p_{\mathsf{succ}}^{\mathsf{onechal}}$ (and this is independent of the hash-rate), so we may safely disregard the impact of this strategy. See Remark 4 for additional details.

Let Q_{cpu} denote the fastest hash rate achievable in an enclave and consider an adversary that uses a network of m distinct enclave-enabled CPUs. The following observation characterizes the adversarial prover's success.

Observation 3. *Let $Q_{\text{adv}} = mQ_{\mathsf{cpu}}$ and $p_{\mathsf{succ}}^{\text{adv}}$ be the probability of an adversarial prover* adv *succeeding in mining a solution π successfully in all the n rounds. From Lemma 1:*

$$p_{\mathsf{succ}}^{\text{adv}} = \left(1-\left(1-\frac{1}{d}\right)^{k_{\text{adv}}d}\right)^{n} \leq \left(1-e^{-k_{\text{adv}}}\left(1-\frac{k_{\text{adv}}}{d}\right)\right)^{n}, \quad (3)$$

where $k_{\text{adv}} = \tau Q_{\text{adv}}/d$.

Thus, to ensure that an adversary cannot successfully compute π in a network of m enclaves, we need to ensure that $k_{\text{adv}} = (\tau m Q_{\mathsf{cpu}})/d$ is small and n is sufficiently large.

Remark 4 (Network of machines with single-challenge ASICs.)**.** A sophisticated strategy that the adversary might attempt in order to bypass our protocol and encumber the key in a TEE is to utilize an outsourced network of machines, each equipped with an ASIC, in such a way that each ASIC is given only one challenge to solve. This ensures that no machine gets access to two challenges that would enable extraction of the witness. This strategy is highly impractical however since the adversary will be required to make strong non-collusion assumptions on the outsourced network. In particular, if any two machines belong to the same entity or collude, they can reconstruct the witness which the adversary needs to avoid.

3) Zero-knowledge

Similar to [35], the prover only submits one Σ-protocol transcript for each commitment, and thus maintains the zero-knowledge property similarly.

D. Practical Security Parameters for ASIC-ZKPoCK

As Observations 1 and 3 show, achieving both completeness and forced revelation introduces a tension in the tuning of d and τ. For completeness, ASIC-ZKPoCK requires moderately large k. To ensure forced extraction, however—specifically, to rule out use of CPUs by an adversary—requires small k_{adv}.

To understand this tension, it is helpful to consider the ratio $N = Q_{\mathsf{asic}}/Q_{\mathsf{cpu}}$, i.e., the speed advantage conferred by an ASIC over a CPU. Given an adversary with a network of m CPUs, the ratio $k/k_{\mathsf{adv}} = (\tau Q_{\mathsf{asic}}/d)/(\tau n Q_{\mathsf{cpu}}/d) = N/m$. Therefore, a secure parameterization requires $m \ll N$, i.e., that an adversary cannot feasibly come close to approaching ASIC speeds with a network of CPUs.

We now show that such parameterization is possible in practical settings.

Estimating Q_{asic} and Q_{cpu}. A top-of-the-line mining ASIC for Bitcoin, the Antminer S19 Pro Hydro, released in 2022, has a rated performance of 154 TH/s [15], i.e., about $Q_{\mathsf{asic}} \approx 2^{47}$ H/s. Each hash in this case is a "Bitcoin" hash: a double invocation of SHA-256 on two 64-byte input blocks, as required for Bitcoin mining. The set $\mathfrak{R}$ of satisfying resources for our formalism will therefore consist of all ASICs with a hash rate of at least Q_{asic}.

Even under optimistic assumptions (including use of native hardware support for SHA), an SGX application[9] on a state-of-the-art 4.60 GHz Intel processor can execute at most about 72 MH/s, i.e, $Q_{\mathsf{cpu}} \approx 2^{26}$ H/s (where hashes here are "Bitcoin" hashes) [52].

Example practical parameterization. Given $Q_{\mathsf{asic}} \approx 2^{47}$ H/s and $Q_{\mathsf{cpu}} \approx 2^{26}$ H/s, we might for instance aim to set $k_{\mathsf{asic}} = 35$, to ensure a prover verification (completness) failure probability $< 10^{-15}$ for a single round of execution. By Observation 1, this probability could be achieved, for instance, by setting $d = Q_{\mathsf{asic}}/7 \approx 2 \times 10^{13}$ and $\tau = 5s$.

With this parameterization, $k_{\text{adv}} = mQ_{\mathsf{cpu}}/d \approx m/300,000$. Consequently, by Observation 3, an adversary with a network of 10,000 CPUs would result in $p_{\mathsf{succ}}^{\text{adv}} < 0.033$

[9]Note that secure use of SGX requires disablement of hyperthreading.

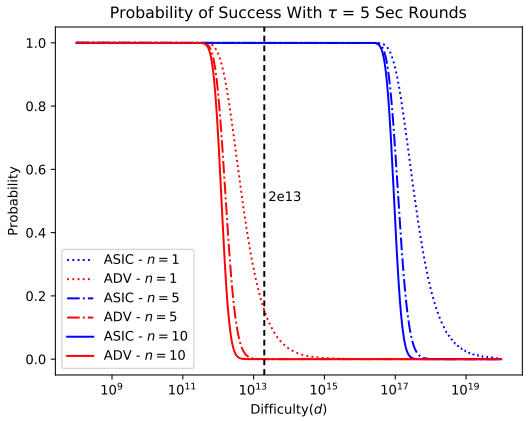

Fig. 5: ASIC-ZKPoCK allows for practical parameters to achieve overwhelming probability of completeness by an ASIC (hashrate 154 TH/s) and negligible probability of success by an adversary with 10,000 state-of-the-art CPUs (hashrate 72 MH/s). n denotes number of rounds. The black line represents a parameterization of $d = 2^{13}$, for which adversarial success probability is negligible but that of ASIC is close to 1.

in a single round of execution of ASIC-ZKPoCK. Executing for $n = 10$ rounds then results in $p^{\text{adv}}_{\text{succ}} \approx 10^{-15}$. Note that, the range of nonce (security parameter β) can be set appropriately to minimize $p^{\text{onechal}}_{\text{succ}}$. For example, by Observation 2, a nonce of length 4 bytes ($\beta = 2^{32}$) would give $p^{\text{onechal}}_{\text{succ}} \approx 10^{-37}$. Figure 5 shows the completeness probability of the ASIC and the success probability $p^{\text{adv}}_{\text{succ}}$ of an adversary with 10,000 CPUs as a function of the difficulty and number of rounds (each round is set to 5 secs).

With a total execution time of 50s, it is possible to achieve an overwhelming probability of completeness as well as an overwhelming probability of failure for an adversary trying to bypass forced revelation with a network of 10,000 CPUs.

Concrete implementation parameters. As we will explain in more detail in Section VII, we implement the ASIC-ZKPoCK prover using an outmoded Antminer S9 ASIC and the verifier using an Ethereum Smart Contract. While this ASIC has a smaller hashrate compared to the latest hardware in the market, and the Ethereum network inherently has a coarse granularity for measuring time, we can still achieve reasonable error probabilities for completeness and forced revelation. Below is one such example parameterization for our implementation:

$Q_{\text{asic}} = 13\text{TH/sec} \sim 2^{43.5}$

$\tau = 12$ sec (Ethereum inter block time)

$k = 12$ so that completeness error for one round is $< e^{-12}$

$d = \frac{\tau * Q}{k} = Q = 13 \times 10^{12}$.

$\beta = 2^{40}$ (Set nonce size to 5 bytes). Notice that the nonce range is exhausted by our ASIC in $2^{40}/Q = 0.08$ seconds, so we queue up new work to the ASIC (with a new challenge) every 0.08 seconds.

Forced revelation error: $p^{\text{onechal}}_{\text{succ}} < 1 - \left(1 - \frac{1}{d}\right)^{\beta} \leq 8.2\%$. We can set the number of rounds n to 5 so that the forced revelation error $\approx 3.5 \times 10^{-6}$. Note that by Observation 3, the probability $p^{\text{adv}}_{\text{succ}}$ of an adversary with 10,000 CPUs each with hashrate $Q_{\text{cpu}} \approx 2^{26} H/s$ succeeding is $< 4 \times 10^{-7}$.

Smart Contract	LOC	Operation	Gas cost
ASIC-ZKPoCK Verification Contract	140	Register new job	366,485
		Initiate challenge	70,209
		Verify proof	7,620,401
CK Registry	130	Record a new proof	54,260

Fig. 6: Gas costs of various calls and Lines of Code in SMACK's smart contracts. At the time of writing, 100,000 gas costs approximately USD 1.3.

Remark 5 (Usage of sequential functions)**.** Our usage of hash-based PoW mining comes with some unfortunate consequences: since evaluation is embarrassingly parallel, we need to rely on assumptions on the parallel processing capabilities (e.g., number of machines) available to the adversary.

A natural question towards removing this constraint is whether we can leverage sequential computation (e.g., through VDFs) instead of parallelizable hash computations. This turns out to be somewhat tricky however since intermediate values may first be computed in a TEE following which the rest of the computation can be done in faster untrusted hardware. We leave the exploration of this direction to future work.

VII. SMACK: An End-to-End CK Implementation

To demonstrate CK proofs in a practical setting, we prototype SMACK (*SMArt-contract enabled CK*)—a *complete, end-to-end* CK system on Ethereum.

SMACK offers a good proof of concept of CK practicality for two reasons. First, smart contracts are highly resource constrained, with limited, expensive computational power, coarse-grained, approximate measurement of time, and no ability to maintain secret state. Making CK proofs work in this austere environment strongly evidences their general practicality. Second, deployment of SMACK in Ethereum has the benefit of supporting a wide variety of blockchain-based services, e.g., voting, Atomic NFTs, enforced NFT royalties, etc., as described in Section II.

We report the gas costs and lines of code (LOC) for these contracts in Figure 6.

SMACK allows for any desired CK method to be used. To demonstrate the practicality of the CK approaches explored here, we implement two CK variants in SMACK: ASIC-ZKPoCK and a TEE-based CK proof system for Android devices that we call *lightweight CK* and describe in Appendix B-A.

We first describe the global architecture of our system in Section VII-A We then describe *CK Registry* component of SMACK (in Section VII-B). Finally, we describe the details of our ASIC-ZKPoCK implementation in Section VII-C, which is the more technically challenging and intricate of the two currently supported CK methods in SMACK.

A. Global Architecture

SMACK consists of different verification contracts, one for each type of CK proof method that is supported. For a given

public PoK value x, the prover supplies a proof (potentially interactively) according to a certain CK method to the corresponding verification contract. The verification contract stores a boolean mapping from x, indicating whether the proof has been successfully verified. This mapping is leveraged by the CK Registry to provide a uniform and well managed interface to application developers.

B. CK Registry

In Ethereum, public keys are associated with *addresses*. SMACK supports CK for the private keys associated with Ethereum addresses.

CK proofs have an important property: Once one has been generated for a given witness / address, it remains *indefinitely valid*. That is because once a private key sk has been exposed to $\mathcal{E}$, the fact of exposure remains true for all time.

SMACK therefore includes a smart contract, called the *CK Registry*, that maintains a permanent record of addresses for which valid CK proofs have been provided.

The CK Registry (deployed at 0x25B270...eE3966) includes a function that maps Ethereum addresses to the type(s) of CK proofs, if any, that have been verified successfully. When a user wishes to submit a proof of complete knowledge for their Ethereum address, it sends the proof to a verification contract (such as the one we describe in Section VII-C) that the CK Registry trusts and then asks the CK Registry to record the event. Applications can then cheaply query the CK Registry to see whether a CK address is verified rather than handling proofs themselves.

The CK Registry is designed to allow the addition of more CK verification contracts in the future, possibly incorporating new classes of CK proofs as they are designed, and likewise to remove existing CK verification contracts (in case the parameters are deemed insecure in the future). As a demonstration of our work, we have used the CK Registry and verification contracts to successfully mint Atomic NFTs for CK addresses. The Atomic NFT contract is deployed at 0xed7B4C...Ba62FF.

C. ASIC-ZKPoCK *implementation*

We use an off-the-shelf Bitcoin miner, the Bitmain Antminer S9, with a hashrate of 13TH/s. Note that while this hardware is not energy-efficient enough to mine Bitcoin competitively at this point, it is sufficient to achieve low error bounds in our scheme. It demonstrates that outmoded hardware can be repurposed successfully for ASIC-ZKPoCK. (It cost us only 200 USD.) We implement the verifier as an Ethereum smart contract, which is deployed at the address 0xAC86fD...B39f4b. All the source code and scripts can be found at https://github.com/CK-anon/SMACK.

System Architecture. Figure 7 shows our system architecture. The prover communicates with the ASIC miner using an open source pool mining software which implements the standard Stratum V2 protocol [4] used for allocating work to the miner, and fetching the PoW solution. The miner does some sanity checks on the input block data (e.g. the block height should be increasing, fields should be the right size

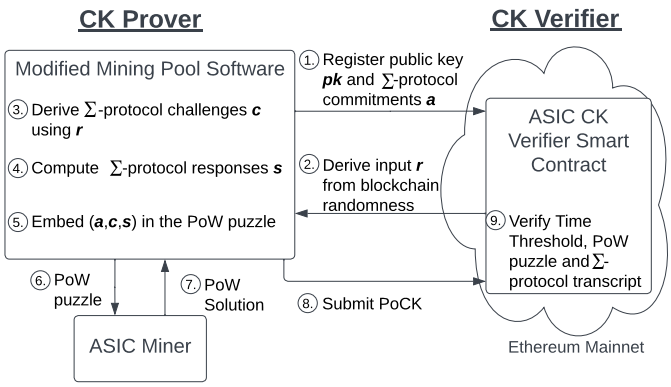

Fig. 7: ASIC-ZKPoCK system architecture. The encircled numbers correspond to steps in a protocol execution. Our CK Prover consists of a (modified) pool mining software and an ASIC miner. Our CK Verifier is implemented as a Smart Contract, deployed on the Ethereum mainnet.

etc.). Therefore, our prover makes use of a private Bitcoin network to generate valid PoW puzzles for the miner. This also allows us to configure the difficulty of the PoW puzzles. We use Schnorr's protocol [58] for our underlying PoK Σ-protocol. While our verification smart contract is used for verifying complete knowledge of the private keys of Ethereum addresses, it can be used for any general PoK value x. To generate the verfier's random challenge r, our smart contract uses randomness from the Ethereum proof-of-stake network [5].[10] For pool mining, the Merkle root in the block header is expanded into a special coinbase transaction along with the adjacent Merkle branches. We place s (32 bytes in our case) inside this coinbase transaction which is allowed to carry arbitrary data.

Setting the Nonce range β. For exploring the PoW puzzle solutions, the Bitcoin pool mining protocol allows the miners to try different values for the *nonce* and *extranonce* fields. The nonce field in Bitcoin header is fixed to 4 bytes and has proven to be too small for the mining market. Therefore, extranonce was introduced whose size can be set by the pool software. Thus, for our case where the range is $\beta(> 2^{32})$, we set extranonce to $\lfloor log_2[\beta]/8 - 4 \rfloor$ bytes. Note that B in the proof transcript π denotes the portion of the Bitcoin block header that excludes the extranonce field, i.e. we treat that field as part of the space of possible nonces.

VIII. Conclusion

We have shown a fundamental limitation in traditional proofs of knowledge (PoKs), the fact that they do not actually prove direct knowledge by a prover when the prover may interact with other entities. This gap in the PoK model introduces a range of coercive attacks, many explored in disparate earlier works. We have formalized complete knowledge as a stronger version of proofs of knowledge. The result is a property, as we have shown, that can help in the design of protocols

[10]This randomness is biasable to small extent but is not material.

resistant to coercive attacks. We have shown that there are practical protocols—using TEEs and ASICs—for proofs of complete knowledge. We hope that our work will stimulate the development of new PoCK / ACK constructions and their use in important practical protocols such as e-voting, deniable authentication, lease-resistant credentials, and more.

An interesting open problem is exploring functionalities stronger than CK for enforcing coercion-resistant voting [14], [28], [41]. For example, CK alone, while helpful, does not ensure coercion-resistant voting given adversarial use of a TEE. Placing all voter functionality in a TEE application *could* help ensure coercion-resistance in this setting, but at the cost of an application-specific realization and bloated trusted-computing base. An important research question is whether there are functionalities that at the same time are general-purpose and achieve broader coercion-resistance than CK.

Acknowledgments. We thank Ian Miers and Rafael Pass for numerous insightful discussions during the early stages of this work.

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APPENDIX A
$\mathcal{G}_{\mathsf{SGX}}$ FUNCTIONALITY

We detail the full $\mathcal{G}_{\mathsf{SGX}}$ functionality in Fig. 8.

$\mathcal{G}_{\mathsf{SGX}}[S, \mathsf{Reg}]$

On initialization: $(\mathsf{mpk}, \mathsf{msk}) \leftarrow\!\!\$\ S.\mathsf{kg}(1^\lambda), I \leftarrow \emptyset$.

On receive $\texttt{getpk}^*()$ from some party $\mathcal{M}$:
- Send mpk to $\mathcal{M}$.

On receive $\texttt{install}^*(idx, \mathsf{prog})$ from some $\mathcal{M} \in \mathsf{Reg}$:
- If $\mathcal{M}$ is honest, assert $idx = sid$.
- Generate nonce $eid \in \{0,1\}^\lambda$.
- Store $I[eid, \mathcal{M}] = (idx, \mathsf{prog}, 0)$ and send eid to $\mathcal{P}$.

On receive $\texttt{resume}^*(eid, \mathsf{inp})$ from some $\mathcal{M} \in \mathsf{Reg}$:
- Let $(idx, \mathsf{prog}, \mathsf{mem}) = I[eid, \mathcal{M}]$, abort if not found.
- Compute $(\mathsf{out}, \mathsf{mem}') = \mathsf{prog}(\mathsf{inp}, \mathsf{mem})$.
- Update $I[eid, \mathcal{M}] = (idx, \mathsf{prog}, \mathsf{mem}')$.
- Let $\sigma = S.\mathsf{sign}_{\mathsf{msk}}((idx, eid), \mathsf{prog}, \mathsf{out})$.
- Send (out, σ) to $\mathcal{M}$.

Fig. 8: $\mathcal{G}_{\mathsf{SGX}}$ global functionality from [50]. Starred operations are re-entrant activation points.

APPENDIX B
DEFERRED DETAILS ON SMACK

A. Lightweight CK

A PoCK system will be most useful if it is widely accessible. Any requirement for expensive specialist hardware—such as an SGX-enabled machine or, worse still, a Bitcoin-mining ASIC—could place CK beyond the reach of most users. While users could in principle outsource CK-proof execution, this would require them to entrust their private keys to third-party services, which could create new risks of key compromise.

Increasingly many users, however, *do* in fact own devices with trusted hardware: their mobile phones. Almost all newly manufactured mobile phones come with TEEs: recent Android devices often ship with Trusty TEE, while iOS devices have Apple's Secure Enclave.

To show how these devices can be used to implement CK proofs, we prototype a protocol design that we call *lightweight CK*. The term "lightweight" here reflects two features of our design: (1) It uses common consumer hardware, but (2) embodies a weaker security model than CK variants using SGX or ASIC. As such, lightweight CK is most suitable as a defense-in-depth layer or for applications where the impact of compromise is not high—e.g., Atomic NFTs (see Section II-B).

Android implementation and workflow. We design a simple lightweight CK tool for Android devices that uses the hardware key attestation API [11] to produce lightweight CK proofs for Ethereum addresses. This API provides a hardware-backed assurance of boot integrity and, by extension, an application's integrity. (Apple's iOS analog is its App Attest Service [12], which can support a CK tool like the one we've implemented for Android.)

Our application itself is simple from a user's standpoint: the user enters a private key sk—exported from a crypto wallet—into a text field[11] and taps a button to generate a TEE key. The application copies the necessary CK proof, described below, to the Android clipboard for the user to paste into a dApp that creates a transaction to a CK verifier smart contract.

The app creates an attestation challenge for the freshly generated TEE key through the key attestation API, and the TEE signs a new certificate containing the challenge. The attestation challenge contains the signature of a static message

[11]Great care will be required in guiding users, as malicious software could dupe unsophisticated users into revealing their private keys. This can be mitigated if common wallets natively implement lightweight CK.

("Android CK Verification"), including an Ethereum message prefix [16] signed by the user's private key. While not strictly needed for a CK proof, the signature serves as a hedge against device compromise. Even if a compromised Android device could generate a seemingly valid integrity verdict, it cannot do so for an arbitrary pk—cooperation from the holder of the corresponding sk would be required.

The API then returns a certificate chain from the new TEE key containing the challenge to the TEE itself to the device manufacturer's certificate authority and, finally, to a root of trust—the Google Hardware Attestation Root Certificate [7]. Within the new TEE key's certificate is an attestation to the integrity of the operating system running on the device as well as a hash of the signing certificates of the application that made the request.

The certificate chain includes all the necessary information to create a CK proof, so submitting a lightweight CK proof to the Android CK Verifier smart contract involves creating a transaction that includes the intended prover address and the complete certificate chain to some root of trust. To ensure the authenticity of a lightweight proof, the contract first checks that the newly created certificate describes an adequately protected Android operating system: a verified boot from a trusted state and a key attestation for an app that matches the package name and signing key of our app. It also checks the attestation challenge embedded within the certificate to verify that sk signed the message. Next, it validates the certificate chain to a root certificate that the verifier trusts by verifying the signatures of each certificate, each one signed by the next in the chain. If everything passes validation, a record of the proof is created in the contract for use by the CK Registry. The cost of verification is approximately 1.5 million gas per certificate.

Signed messages cannot protect against a compromised device producing valid verdicts for others' keys, so we rely on per-device limits to mitigate the effect of a compromised device. Until mobile devices support on-device attestations, integrity measurements of the operating system are the closest way of verifying that the application ran as intended and a complete private key was entered into the device.

Limitations. Key attestation appears not to be foolproof. For example, there have been reports of a broken TEE keystore implementation in the ASUS ROG 3, compromising system integrity and still allowing a "strong" hardware-based integrity check to pass irrespective of bootloader status [2]. Google itself recommends a defense-in-depth approach, with attestation services as only one of several signals of abuse. To mitigate the problems caused by an entire class of devices containing faulty TEEs, the Android CK Verifier contract allows individual CA certificates to be revoked or trusted.

Privacy. Publishing a complete TEE certificate chain to a public blockchain comes with its own privacy issues. Each TEE certificate must be signed by a device manufacturer's public key for the certificate chain to be complete, which means that device manufacturers could easily associate the Ethereum address of a lightweight CK participant with the mobile device used to create the TEE certificate chain. This is because the Ethereum address being verified is contained within the TEE-signed certificate, and device manufacturers can associate a TEE's public key with its corresponding device both during manufacturing and whenever the device requires an updated certificate chain. As a point of reference, the first intermediate certificate of our sample device expires on a monthly basis, so at least once a month, the device must contact its manufacturer for a fresh intermediate certificate.

In order to prevent this type of privacy leak, rather than submitting the certificate chain directly to a smart contract, the certificate chain could instead be submitted to an application running within an off-device attesting TEE, such as Intel SGX. The attesting TEE application verifies the most sensitive part of the chain—including the mobile device TEE's public key—and the integrity state of the mobile device embedded inside the leaf certificate. The smart contract would then only need to verify the application's attestation to establish whether the lightweight CK attempt was successful, thereby keeping the end of the certificate chain from being disclosed. Then, collusion between both the mobile device manufacturer and the organization hosting the off-device TEE, as well as a feasible attack on the off-device TEE itself, would be necessary to deanonymize accounts. An attack on the TEE could reveal the tail of the certificate chain which, when revealed to the device manufacturer, could be mapped to a device.

In an alternative to the TEE-based privacy approach, the prover's CK proof could be a redacted certificate chain along with a ZK proof (preferably a *succinct* such proof) of the correctness of the redacted part of the chain.

Target applications. Google's Play Integrity API is currently the more commonly used method for developing assurance of application integrity inside Android apps, and its "strong" category of integrity verdicts include similar hardware-backed key attestations from a TEE. Although its integrity verdicts can range in strength from basic compatibility tests to hardware-backed key attestation, it has seen wide use by consumer services (e.g., Netflix), mobile games (e.g., Pokémon Go), banking applications, etc. for application integrity [44].

Given the security limitations discussed above, lightweight CK is most suitable as a defense-in-depth layer or where the impact of compromise is limited. For example, if CK is compromised for an Atomic NFT, that NFT can be fractionalized: an undesirable but not catastrophic outcome. The same is true of key-coupling for royalty payments. In contrast, only strong CK would meet the levels of security envisioned for soulbound tokens [65], which are identity documents.

Through the CK Registry, individual applications can support the specific CK proof types that match their security models.